- **Lesson 36- Critical Numbers**
  -A critical number is where the derivative is equal to zero or undefined
  -Local Extrema: a point that is either a local maximum or minimum
    - ➔ Not all critical points are local extrema, but local extrema is a critical point

  -<u>Example:</u> Find all critical numbers for f(x)= $x^3 + \frac{3}{2}x^2 - 6x$

  Step 1- Take derivative: f'(x)= 3 x^2 + 3 x - 6

  Step 2- Set derivative equal to zero: 3(x^2 + x - 2)=0

  (x+2)(x-1)=0

  Step 3- Find x-values where derivative is equal to zero

  Answer: x= -2, 1

- **Lesson 37- Chain Rule**
  -$\frac{d}{dx}$ [f(g(x))]= f'(g(x)) $\cdot$ g'(x)

  -<u>Example:</u> Find f'(x) of f(x)= $(x^2+4)^{12}$

  f'(x)= 12$(x^2+4)^{11}$ $\cdot$ 2x

  Answer:f'(x)= 24x$(x^2+4)^{11}$

  -<u>Example:</u> Find f'(x) given f(x)= $sin^5(x^2)$

  Remember that $sin^5(x^2)$ = (sin(x^2))^5

  f'(x)= 5(sin(x^2))^4 $\cdot$ cos(x^2) $\cdot$ 2x

  Answer: f'(x)= 10x (sin(x^2))^4 $\cdot$ cos(x^2)

- **Lesson 38- Integrals of Sums**

- Recall:

  $\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$

  - The same rule applies with integrals:

$$\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

- Example: $\int (2x+e^x+3\cos(x))\,dx$

$=\int 2x\,dx + \int e^x\,dx + \int 3\cos(x)\,dx = (x^2+c_1)+(e^x+c_2)+(3\sin(x)+c_3)$

$=$ $\mathbf{x^2+e^x+3\sin(x)+c}$

- Example: $\int 4t^{-1}+3t^{-1}+4\cos(t)\,dt$

$=\int 4t^{-1}\,dt + \int 3t^{-1}\,dt + \int 4\cos(t)\,dt = (4\ln|t|+c)+(3\ln|t|+c_1)+(4\sin(t)+c_2)$

$= 4\ln|t|+3\ln|t|+4\sin(t)+c =$ $\mathbf{7\ln|t|+4\sin(t)+c}$

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- **Lesson 29 and 31- Differentials and Product Rule**

-Relationship between the function, first derivative, and second derivative

- ➔ If the function is increasing the derivative is positive
- ➔ If the function is decreasing the derivative is negative
- ➔ If the function is concave up the derivative is increasing and the second derivative is positive
- ➔ If the function is concave down the derivative is decreasing and the second derivative is negative

- Product Rule

$$\frac{d}{dx}(f(x) \cdot g(x)) = (f'(x) \cdot g(x)) + (g'(x) \cdot f(x))$$

-<u>Example:</u> $f(x)= 3xy^3+2x+4y+x^2y$ find the derivative

Answer: $f'(x)= 9xy^2+3y^3+2+4+2xy+x^2$

- **Lesson 32- Indefinite Integrals**
  -Indefinite integrals give you a family of functions starting with the derivative of that family of functions

  -Example: $\int 4x^3\ dx = x^4 + c$

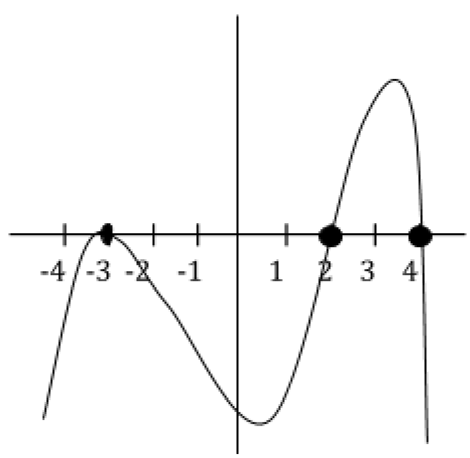
  To check your answer take the derivative of your answer so
  f(x)= x^4 +c
  f'(x)= $4x^3$
  Since the derivative matches the inside of the integral we know that our answer is correct

- **Lesson 33- Graphing Polynomial Functions**
  -Let's say we have a function f(x)= $(x-a)n\ (x+b)m\ (x-c)p$, n,m, and p are all exponents
  - ➔ if n is even then at the zero x= a the line will bounce
  - ➔ if it is n is odd then the line will pass through

  - The end behavior of the function is determined by the total of the exponents and the sign of the leading coefficient
  - ➔ If the total of the exponents is even then the end behavior will look that of a parabola
    - ◆ If the leading coefficient is negative the end behavior will be that of an upside down parabola
    - ◆ If the leading coefficient is positive that end behavior will be that of a positive parabola
  - ➔ If the total of the exponents is odd then the end behavior will be increasing on one end and decreasing on the other so like ↑↓ or ↓↑
    - ◆ If the leading coefficient is negative the end behavior will be ↑↓
    - ◆ If the leading coefficient is positive the end behavior will be ↓↑

  -Example: Graph f(x)= $-3(x-2)^3(x+3)^2(x-4)$

  Zero's: 2, -3, 4
  Passes Through: 2, 4
  Bumps Off: -3
  End Behavior: ↓↓



- **Lesson 24- Power Rule**

  - <u>Example</u>: $f(x)=x^3+3x^2+3$

Bring down the power and subtract one, remember to use correct notation (f'(x) or dy/dx, etc.

Answer: $f'(x)= 3x^2+6x$

- **Lesson 27- Tangent Lines and Higher Order Derivatives**

  - Given a function f(x) that is differentiable then the equation to the line tangent at x=a is y= f'(a)(x-a)+f(a)
    - ➔ Slope = The derivative of f(x) at a so f'(a)
    - ➔ Point= (a, f(a))

-<u>Example</u>: Find The equation of the line tangent to the graph of $f(x)=3x^3+x^2+2x$ when x=4

First find the slope

$f'(x)= 9x^2+2x+2$ f'(4)= 154

Next find the point

$f(4)= 3(4)^3+(4)^2+2(4)= 216$

Answer: y= 154(x-4)+216

- Second Derivatives

Take the derivative of the first derivative

-<u>Example:</u> Find the second derivative for $f(x)=x^3+3x^2+3$

$f'(x)= 3x^2+6x$

Answer: f''(x)= 6x+6

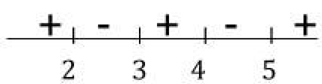
- **Lesson 28- Graphs of Rational Functions**

  - If $R(x)=\frac{k(x-r)(x-r2)...(x-rn)}{k\,(x-q)(x-q2)...(x-qm)}$

    - ➔ Zeros of R(x) are the zeros of the numerator
    - ➔ Vertical Asymptotes (VA) are zeros of the denominator
    - ➔ Hole is when a zero appear is both the numerator and denominator

  -<u>Example:</u> Graph by hand $R(x)=\frac{(x-4)(x+3)(x+2)}{(x+3)(x-5)(x-3)}$

1.) Holes? x= -3
2.) VA's? x= 5, 3
3.) Zero's? x= 4, 2
4.) Sign Chart (consists of both zeros and VAs)



To find at what point the whole is plug x= -3 into R(x) , R(-3)= 72
Answer: